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Total Pressure Recovery for a Dump Combustor

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Nomenclature

A	= area
k	= ratio of specific heats
M	= Mach number
p, p_T	= pressure and total pressure, respectively
R	= universal gas constant
T, T_T	= temperature and total temperature, respectively
w	= flow rate
$\Delta T/T$	= $(T_{T_4} - 1)/T_{T_3}$
η_p	= combustor pressure efficiency

Subscripts

2	= diffuser exit
3	= combustor entrance
4	= combustor exit
5	= nozzle throat

Introduction

THE over-all total pressure recovery for a combustor can be obtained by combining the total pressure recovery across the combustor entrance and the total pressure recovery across the combustor. These two pressure recoveries are generally determined by separate methods. A method has been developed from one-dimensional gasdynamics which predicts the over-all total pressure recovery for a combustor within 2% of experimental results. The method is applicable for combustors in which the incoming flow undergoes a rather sudden expansion. Such combustors are referred to as dump combustors and are used in after-burners and ramjet engines. A schematic of a dump combustor with station notation is shown in Fig. 1.

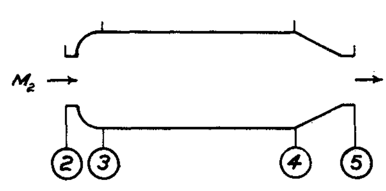

Fig. 1 Schematic of dump combustor.

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Development of Total Pressure Recovery Relation

There are two main sources of total pressure loss in a dump combustor. The first is viscous losses associated with the sudden expansion of the flow across the dump or combustor entrance. The second is fluid acceleration as a result of the heat addition.

The pressure recovery across the dump p_{T_3}/p_{T_2} will be determined by analyzing the dump as a sudden enlargement, as is done in pipeline flow. References 1 and 2 show good agreement with experimental results using this approach.

The momentum and continuity equations across a sudden enlargement are

$$p_3 A_3(1+kM_3{}^2) = p_2 A_2(1+kM_2{}^2) + p_2(A_3 - A_2) \tag{1}$$

$$p_3 A_3 M_3[1+(k-1)M_3{}^2/2]^{1/2} = p_2 A_2 M_2[1+(k-1)M_2{}^2/2]^{1/2} = w[RT_{T_2}/k]^{1/2} \tag{2}$$

The perfect gas relations were assumed in writing Eqs. (1) and (2). Combining Eqs. (1) and (2), and using the isentropic relations for total to static properties, one can determine the dump total pressure recovery p_{T_3}/p_{T_2}. The relationship for p_{T_3}/p_{T_2} is presented functionally by Eq. (3) and graphically in Fig. 2.

$$p_{T_3}/p_{T_2} = f(M_2, A_3/A_2, k) \tag{3}$$

The total pressure losses across the combustor are due to heat addition and friction. The combined effect of heat addition and friction can be analyzed for a compressible fluid using the methods outlined in Ref. 3. The frictional losses in a flow-through combustor are generally small and will be neglected in this analysis. The combustor pressure recovery p_{T_4}/p_{T_3} will be predicted from the Rayleigh line relationships, which are

$$p_{T_4}/p_{T_3} = \{(1+kM_3{}^2)/(1+kM_4{}^2)\} \times \{[1+(k-1)M_4{}^2/2]/[1+(k-1)M_3{}^2/2]\}^{k/(k-1)} \tag{4}$$

$$\begin{aligned}\Delta T/T &= (T_{T_4}/T_{T_3}) - 1\\ &= \{[M_4{}^2(1+kM_3{}^2)^2]/[M_3{}^2(1+kM_4{}^2)^2]\} \times \{[1+(k-1)M_4{}^2/2]/[1+(k-1)M_3{}^2/2]\} - 1\end{aligned} \tag{5}$$

It can be seen from Eqs. (4) and (5) for a given M_4 that, as the temperature rise term $\Delta T/T$ increases, M_3 and p_{T_4}/p_{T_3} decrease. The relationship between p_{T_4}/p_{T_3} and $\Delta T/T$ is presented in Fig. 3. From Fig. 2, it can be seen that, as M_3 decreases, the dump pressure recovery p_{T_3}/p_{T_2} increases. Therefore, opposing effects exist across the entire combustor with the addition of heat—as $\Delta T/T$ increases, p_{T_4}/p_{T_3} decreases, and p_{T_3}/p_{T_2} increases, and vice versa.

These opposing effects have a compensating effect on the over-all combustor pressure recovery and result in p_{T_4}/p_{T_2} being fairly constant for a wide range of heat additions and/or fuel air ratios. The limiting or maximum value from p_{T_4}/p_{T_2} is the

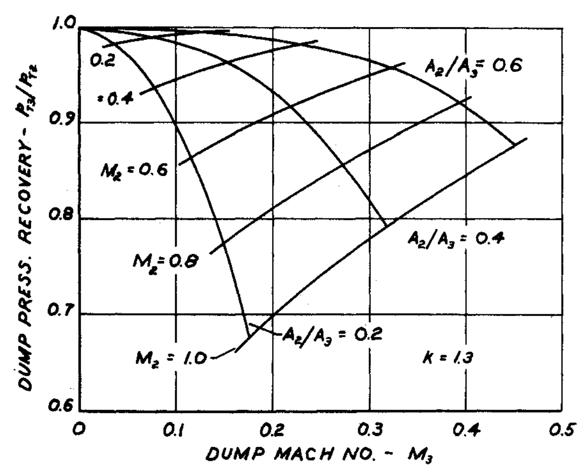

Fig. 2 Dump total pressure recovery.

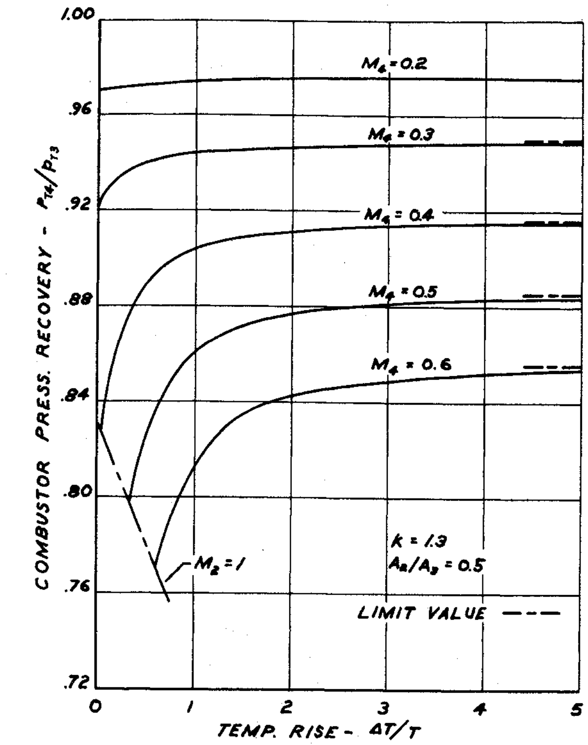

Fig. 3 Combustor total pressure recovery.

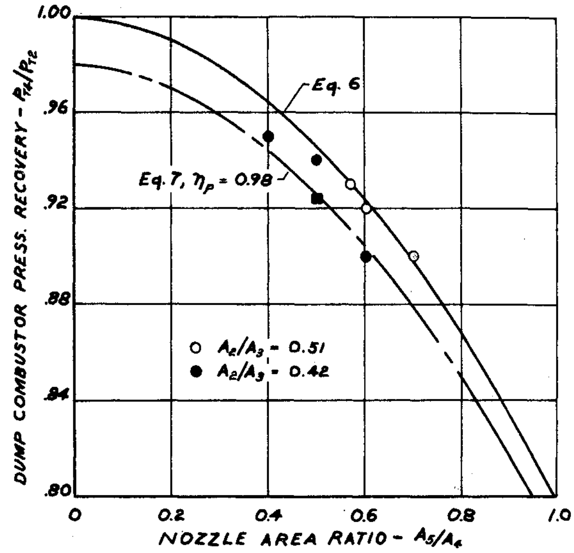

Fig. 4 Over-all combustor total pressure recovery.

Rayleigh line pressure recovery for infinite heat addition, which can be obtained by setting $M_3 = 0$ in Eq. (4).

$$(p_{T_4}/p_{T_2})_{\max} = [1+(k-1)M_4{}^2/2]^{k/(k-1)}/[1+kM_4{}^2] \qquad (6)$$

This limiting value is noted on Fig. 3.

For a combustor operating with a choked constant area nozzle, M_4 is constant and independent of the combustor inlet conditions and fuel air ratios. M_4 is a function only of the exit nozzle area ratio A_5/A_4. The maximum combustor pressure recovery as defined by Eq. (6) is presented in Fig. 4 as a function of A_5/A_4.

Experimental data in Ref. 1 indicate that the over-all combustor pressure recovery p_{T_4}/p_{T_2} is independent of the combustor inlet conditions and fuel air ratio. The experimental values for p_{T_4}/p_{T_2} from Ref. 1 are presented in Fig. 4 for various exit nozzle area ratios. The data points presented in Fig. 4 represent an average of the measured pressure recovery over a range of fuel air ratios. The open points are for a coaxial dump and the closed points are for a 30° dump angle. The dump area ratios A_2/A_3 for the two combustors are noted on Fig. 4. The square point in Fig. 4 was determined from an empirical equation for the pressure drop across a turbojet combustor. The area ratio used in plotting this point is the turbine flow area to the total combustor flow area.

The agreement between experimental data and Eq. (6) in Fig. 4 is within 2%, which is within the accuracy of the experimental data. The degree of agreement is even more remarkable when one considers that the experimental data are for combustors with different geometries.

Closer agreement can be obtained by multiplying Eq. (6) by a combustor pressure efficiency η_p, which is defined by Eq. (7).

$$\eta_p = (p_{T_4}/p_{T_2})/(p_{T_4}/p_{T_2})_{\max} \qquad (7)$$

Experimental data, such as presented in Fig. 4, can be used to determine the value of η_p for a given combustor or a class of combustors. The combustor pressure efficiency η_p will account for frictional losses during the heat addition process which have so far been neglected. A curve for $\eta_p = 0.98$ is included in Fig. 4.

The advantage of the method, as described by Eqs. (6) and (7) for determining the combustor pressure recovery p_{T_4}/p_{T_2}, is that only one variable, A_5/A_4, is required to obtain a good estimate. Use of this method will greatly simplify the mathematic modeling of combustor performance. The use of this method should also reduce the required amount of combustor pressure recovery testing.

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Performance Testing of a Transit Generator at JPL

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Introduction

A TRANSIT spacecraft, launched on September 2, 1972 into Earth orbit, was powered by a radioisotope thermoelectric generator (RTG). After approximately one month of operation in space, the loss of telemetry data precluded all verification of the RTG operative behavior.[2] In March 1973 a Transit-type generator Model QM-3 activated by radiant heat from an electrically heated source (ETG) was delivered to JPL for long-term parametric and life tests.

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Index category: Spacecraft Electrical Power Systems.

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